

## AGHA AND MATH

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The story that follows gives another point of view on the logarithm business. The author, Vladimir Karapatoff, was an electrical engineer, who taught in several Russian colleges and consulted for the Russian government before coming to the United States in 1903. In the United States he served as a consultant for various industries and taught electrical engineering at Cornell University from 1904 to 1939.

Dr. Karapatoff invented and patented several electrical devices, and also a five-stringed cello, which he played in public. He played the piano well enough to give public performances and wrote a book of poetry, *Rhythmical Tales of Stormy Years* (1937).

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Once there lived in an oriental country a rich landed proprietor. He had a long string of names—first, middle, and last—each in honor of an ancestor, but to those about him he was simply known as Agha, the Master. He had numerous slaves who raised crops for him, took care of his orchards, and looked after his livestock. Only two out of his wives were living, and among his children there was a pretty girl, Rhia, of marriageable age. In fact, a neighboring landed proprietor, Hussein, wanted to marry her, but

he already had five wives which was the maximum number allowed by the law. He had to wait until one of them died or could be disposed of as a present to one of his hangers-on. Of course, he could take her as a harem woman, or, using a Navy expression, wife's mate, Third Class, but Agha was too proud to consent to such an arrangement. The city of Khaleb, situated on a large navigable river, was just half a day's journey from Agha's estate, by oxcart, and his produce and cattle were occasionally taken there for sale on market days.

It was this disposal of his farm products and the buying of spices, silks, and a few other items that were among Agha's principal headaches. He had some faithful slaves and skilled supervisors, but it was difficult to find a man who could keep books, enter the sales and the purchases, and know how to figure out the amount of supplies on hand, and to tell Agha if he was making money, and if so how much.

Now Agha was a thrifty man, in spite of his great wealth, and always insisted on accurate bookkeeping and accounting. For example, if he sent to the market seventeen calves, to be sold at eleven silverpieces each, he wanted to know what the total amount would be. Each of his supervisors and some intelligent slaves always had to carry at their belts leather bags filled with pebbles. In the above case, the slave to be sent with the calves had to put on a table, side by side, seventeen piles of pebbles, eleven pebbles in each pile, and then count the total number of pebbles. As a precaution, another slave was ordered to do the same in a different room, and then each reported his findings to Agha. If there was a discrepancy, each was given ten lashes at the whipping post, and two other slaves were called to

do the addition anew, until the amount was settled. Then those who got the right amount (or at least checked each other) had to give twenty lashes to the others. This they were always glad to do, having received undeserved punishment before. This is how the old saying originated: "Accuracy is in your hands, inaccuracy on your back." Later the proverb was abridged to the terse saying: "Inaccuracy hurts." Anyway this problem of correct accounts was on Agha's mind most of the time, and he kept his eye open for an improvement in the situation.

One day Agha had several carts with produce and cattle at the Khaleb market, and he was strolling among the vendors surveying the situation and watching his slaves and other merchants. He noticed a young man, whom he had never seen before, approaching a vendor and after a brief conference taking out a peculiar contraption from a bag, doing something with this contraption, whispering something to the vendor, and then replacing the contraption in the bag. Whereupon the vendor gave the young man a copper coin and the young man thanked him and went to the next vendor. Some accepted his services, others shook their heads negatively. Agha was naturally a curious person, and in this case he wanted to be sure that nothing was being done to put him at a disadvantage in selling his wares. So he followed the young man and finally asked him what he was doing. "Oh," said the young man in a pleasant lisp, "I am Math, the Abacus, and I figure out the amounts of sales for my clients. May I therve you thir? My charges are very reasonable and the accuracy is guaranteed."

"I saw you take a gadget out of that bag and use it. May I see it?" asked Agha.

"Sorry, thir," said Math the Abacus hastily, "this device

is my stock in trade and I keep it partly covered even when I am using it."

"Listen, Math. I am Agha, the well-known landed proprietor up the river and I can make your future if your judgment is as good as your skill is supposed to be. I want to talk to you tonight, over yonder at Mustapha's coffee house. Be there soon after sunset."

Seated over their cups of fragrant Arabian coffee, Agha said to the young man, "First tell me about yourself and how you happen to be in this town."

"My story is very simple," answered Math. "My father is a small landed proprietor a few miles down the river, and of course we all have heard of you, Agha. My father naturally wanted me to stay on his estate since he is getting old and weak. Besides, he wanted me to marry a neighbor's daughter, Podagra, for whom I do not care at all. I am more of a bookworm, and my ambition is to become an astrologer. This is why I ran away from home and came here hoping to enter the Astrologers' Academy in Khaleb. Unfortunately, they want too much money for instruction, and I also have to have something to live on until I am skilled enough in predicting the course of planets and their influence on human fortunes. . . ." The young man stopped and looked at Agha as if to find out if this information was just what he wanted.

"What is your real name?" asked Agha.

"My name is Massy, or Mass for short, but on account of my lisping which you no doubt have noticed, I have to pronounce it Math, and that's how I got this nickname."

"What about the rest of your name—the Abacus?" asked Agha.

"Oh, this is the name of my computing device which has

come from India and is not known in this country. With it I can figure out large amounts in no time—for example, 245 bushels of grain at 31 coppers a bushel. Of course, I had to develop considerable speed in my fingers, and my brain just naturally takes to figures. On market days I am making good money and I am saving as much as I can to pursue my studies at the Astrologers' Academy." Agha looked over the young man carefully, and with his knowledge of human nature he quickly sized up the young man as honest but rather impractical, though intellectual. His clothes were shabby and torn, his countenance emaciated and it was clear that he was barely eking out his existence.

"Here's my proposition to you, Math the Abacus," said Agha. "I am willing to take you on my estate as an indentured servant for two years. You are to live with my other servants and to be treated as such, including occasional floggings for mistakes and knavery. Your duties will be to keep accounts, train others in doing figures, and find out my profit or loss."

"But I cannot disclose the secrets of my abacus," interrupted Math hastily.

"Damn the abacus," said Agha. "My slaves could not understand it anyway, nor could they develop the skill in their horny fingers to handle it. Just make them count the pebbles quickly and accurately. There is no better way of getting practical results."

"Oh, yes there is," said Math, "by multiplication, instead of addition."

"I had a Greek slave once," Agha told him, "who mentioned multiplication to me. I had to flog him three times before he gave up the idea, and I shall do the same to you."

"But Agha," exclaimed Math, "multiplication is used in the Astrologers' Academy in computing paths of celestial bodies, and it is a much quicker method than your addition of pebbles. . . ."

"You heard me," said Agha sternly, "now listen further. For two years you will just get your keep and a few coppers to spend when I send you to town. But if your work is satisfactory and you prove to be a loyal servant, at the end of two years I shall give you three purses of gold. One will pay for your Academy instruction, one will keep your body and soul together while you are learning to fool and deceive people by gazing at the stars, and the third one will start you on your career. Yes or no?"

"I most gratefully accept your offer, merciful Agha," said the young man avidly, "and I am ready to go with you any time you return to your estate."

Thus it was that Math the Abacus became installed on Agha's estate as bookkeeper, accountant, and teacher of addition. In spite of his superior skill in arithmetic, he was modest and patient with the slaves and helped them all he could in their computations, to save them from the whipping post. He himself was careful not to mention multiplication to his master, although he practiced it secretly in arriving at results on his abacus.

This abacus which he carefully concealed from everyone consisted of a set of wooden sticks with ten beads on each stick. Each bead on the first stick is worth one unit; on the second stick each bead is worth ten units; on the third each bead is worth one hundred, etc. When he wanted to add 3 and 4, he moved three beads to the other end of the stick and then four beads more. The result was 7 beads. When he wanted to add 5 and 7, he set off 5 beads and

then he could not set off 7 more, so he added 5 only, restored the 10, took a bead off the next stick which was worth ten and added 2 more on the first stick, thus getting a result of 12. Of course, he developed such a dexterity of fingers in addition and subtraction that no one could tell by watching his fingers just how he was getting his results. When it came to multiplication, he quickly converted it into addition. For example, if someone in the household wanted to know how much 7 yards of cloth were at 13 coppers per yard, he said to himself, "Three times 7 is 21," and marked off 2 and 1 on the adjacent sticks of the abacus. Then he whispered, "Seven times 10 is 70," and added 7 beads to the previously marked 2. He did all this partly covering the abacus, working by touch, so no one could tell what he was doing. Then he would glance at the beads and say modestly, "I believe it is 91, but you better check it on your pebbles." It goes without saying that the addition on pebbles always confirmed his results. Thus in time all the slaves and supervisors explicitly believed his computations, and his influence gradually grew, the more so as he picked up a little astrology here and there and occasionally predicted rain, the sex of infants about to be born, or a flogging. When his predictions did not come true, it was always the case of a hostile planet or constellation which he hadn't noticed in time to correct his prediction.

A year passed. At first he counted weeks and days when his indenture would end so he could return to Khaleb and enter the Astrologers' Academy, but now he seemed to dread the approach of that day. The simple reason was the two bright stars on the face of the pretty Rhia which sparkled every time he happened to meet her in his routine calls

at the master's house. Of course, he belonged to the same class of society as Agha, and from this point of view was entitled to ask for Rhia's hand, but what chance had he, a poor indentured servant who hoped to become an astrologer? Besides, he had learned from other slaves that Rhia was partly promised to the rich Hussein as soon as his heart would become vacant for a new love. Nevertheless, so strong was Math's love that he took the courage to speak to Agha about it in rather general roundabout terms. But the shrewd landowner interrupted him,

"Are you still thinking of that multiplication, or are you now convinced that addition is the only practical way of business bookkeeping, no matter what those faker astrologers of yours believe?"

Now Math remembered the threat of flogging, and he knew that it was either multiplication or Rhia, so he said modestly, "You are my benefactor, and through you I hope to become an independent astrologer. Your servants and overseers are doing well with pebbles under my direction, so why should I invite trouble for myself?"

"But isn't it true," persisted Agha, "that you are using that cursed multiplication on your abacus?"

"I came here under the condition that my abacus and the method of its use should remain my secret," said Math quietly, "and I am sure that my master will want to keep his part of our bargain."

"I do not care how you use that cursed contraption of yours," said Agha, "but when you leave we shall have to go back to our slow addition on pebbles, with constant floggings and cutting out of manti-issas."

Now manti-issa was an article of food that the slaves were particularly fond of, and when a servant obtained a wrong



result in counting pebbles, he was not only flogged, but deprived of this important article of food for a few weeks, depending on the magnitude of the error committed. This manti-issa consisted of pig's duodenum stuffed with scraps of meat from the master's table, and was practically the only meat dish that the slaves had. Being deprived of manti-issa (which literally means "filled little stomach") was considered almost as severe a punishment as ten lashes, and this made the slaves doubly careful in counting the pebbles.

"I'll tell you what I have in mind," continued Agha. "Whatever that multiplication is, there ought to be some way of doing the same thing by addition. Of course, I do not know how this is to be achieved, but you are supposed to be skilled at figures, and ought to be able to invent a way."

"Does the master have in mind that if I have to take 13 seven times (which makes 91) I could add two numbers instead and still obtain 91?"

"Something of the sort," said Agha carelessly, rising to his feet.

"But merciful Agha," exclaimed the youth in despair, "it is not humanly possible to add two figures and to obtain the same result as by multiplying them. The only exception is 2 plus 2 and 2 times 2. . . ."

"Very well," said Agha quietly, "I understood you to say, or rather to hint, that you would like me to give you my daughter Rhia in marriage. I hereby consent to this marriage provided that you bring me instructions whereby multiplication (that cursed operation which I do not understand) is replaced by plain addition which anyone can understand. The morning after you demonstrate this trick

to my satisfaction Rhia will be yours—and Hussein can keep his five wives.”

Math the Abacus was both crushed and excited. The problem still seemed impossible of solution, but now he could think of Rhia at least as of a possible though remote goal, and perhaps speak to her occasionally, and have her encouragement. All he knew so far was that she smiled responsively when she passed by. Now he dared to speak to her and tell her how the thought of her would inspire him to a superhuman effort to accomplish the impossible. Yes, he succeeded in seeing her alone and he told her what her father demanded as the price of her hand, and she told him how her thought would always be with him, and how he should work and have courage, until they would be united.

Now came long sleepless nights of inquiry and search into numbers and their properties. During the day he was fully occupied with the records and computations of all sorts, and there wasn't that quiet solitude which a genius needs for creative work. The first gleam of hope came to him that there existed in arithmetic something akin to what Agha demanded of him, namely, the addition of powers of ten. “One hundred,” he said to himself, “is ten to the power of two, and one thousand is ten to the power of three. The product of the two is one hundred thousand, or which is the same, ten to the power of five. Thus to multiply ten to the second power by ten to the third power, we simply added the exponents, that is the powers of ten which represented the two multiplicants.”

His logical mind immediately saw the next step. If any number, say 2, could be thought of as a power of ten, then the problem would be solved. What power of 10, 2 was he could not even imagine, so he called it  $x$ , and wrote

$10^x=2$ . Similarly, he said to himself, we would write  $10^y=3$ . Here  $y$  is the unknown power to which 10 must be raised to obtain 3. Assuming this to be possible, the logical consequence would be:

$$2 \text{ times } 3=10^x \text{ times } 10^y=10^{x+y}=6$$

Thus, instead of multiplying 2 by 3, it is only necessary to add  $x$  and  $y$ . Then, if we know what power of 10, 6 was we should have our answer. "Maybe the old Agha isn't so dumb as I thought—in fact, he seems to possess a great mathematical intuition."

With renewed vigor and hope he returned to this problem over and over again as the time of his departure for Khaleb grew closer and closer. Now that he could see Rhia occasionally he wanted time to pass as slowly as possible, the more so since he hoped to solve the mathematical task put before him by his master. He now concentrated on the question of what power of 10 the quantity 2 was. He said to himself, "two lies between 1 and 10. Ten to the power of 1 is 10 and 10 to the power of zero is 1. Hence, 2 must be equal to a power of 10 lying between zero and 1." Yet he still could not see any way of finding the value of this power. So he said to himself, "suppose I assume, as the first approximation, that 2 is 10 to the power of 0.2; 3 is 10 to the power of 0.3, etc.; 9 is 10 to the power of 0.9, and finally 10 is 10 to the power of 1." It did not take him long, however, to discover some inconsistencies in these assumptions. For example, 2 times 3 is 6, so that the exponent of 6 should be the sum of those of 2 and 3. Yet, according to his assumption, the sum of these exponents was 0.5, whereas the exponent of 6 was 0.6. Again 3 times 3 is 9, so if the exponent of 3 is 0.3, the exponent of 9 should be 0.6, whereas he assumed it to be equal to 0.9. By several trials of this sort,

he concluded that the exponents of 2, 3, etc., were larger than he had assumed. So instead of taking them equal to 0.2, 0.3, etc., he now assumed them to be equal to 0.3, 0.4, etc., and the exponent of 9 to be 0.95, instead of 0.9. New trials showed that the new figures gave products closer to the correct values than before. When this moment came, he at once realized that he was on a fair way toward a solution of Agha's problem which instead of being an absurd supposition of an ignorant man was an intuitive flash of a genius, a flash that would make Math's name go to posterity as that of a prominent mathematician who made complicated computations possible in a simple manner.

After another tete-a-tete with Rhia, he now betook himself to finding more accurate figures for those exponents of the first nine digits. His goal was to find such values of these exponents that any two exponents when added gave the exponent of the number which is the product of these two numbers. After several nights, he finally arrived at the following figures:

Nos.	1	2	3	4	5	6	7	8	9	10
exponents of	0.000	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1.000

He then realized that he had to extend this table to include the exponents of all the numbers up to 100. At first the task seemed tremendous, but gradually he recognized that he already had the exponents of a goodly number of quantities greater than 10. For example, the exponent of 20 is 1.301, because it had to be equal to the sum of the exponents of 2 and 10. The exponent of 24 is 1.380, for it had to be equal to the sum of the exponents of 8 and 3, or of 4

and 6. Before he knew it, he had the exponents of all but the prime numbers between 10 and 100. These prime numbers, like 29 or 71, were not numerous and he could guess at their exponents from those of the two adjacent numbers.

Finally he was able to record on a piece of sheepskin the exponents of all the numbers between 1 and 100. His devoted Rhia learned to help him, and secretly checked one multiplication after another, until they could find no combination that did not check. For example, she would take the exponents of 8 and 12 and add them. The result would give the exponent of 96, as it should be.

Then it was a matter of more patience and more figuring to extend the table to 1000. On the night Math finished his table, for the first time in many months, he slept the sleep of a man assured of his future, his fame, and a loving wife.

I wish I could report truthfully that he dreamt of beautiful Rhia, but he was too tired to dream of anything or anybody. The shepherd's pipe awakened him before the sunrise. He jumped up; realization of his luck became clear to him suddenly, and he ran to his master's house. He stopped before Agha's bedroom window and began shouting at the top of his shrill voice, lisping more than ever,

*"Lol Agha, Rhia ith mine!"*

The master woke and naturally assumed that the house was on fire, until he recognized Math's familiar lisp. He told the woman in attendance that night to call the chief supervisor who slept a few rooms away. When he appeared, Agha instructed him to give Math twenty lashes for disturbing the peace. "Put him on bread and water for a

month, and see to it that no manti-issa is slipped to him. After a month I may be willing to hear what he has to say."

Fortunately time was counted in that country by lunar months so that Math's punishment lasted only twenty-eight days instead of thirty. Finally his new system of multiplication by addition was presented to Agha and approved by him as being within the intellectual grasp of the dumbest of his servants and slaves. He received Rhia for his wife and the promised three purses of gold. Hussein was just a day late in palming off one of his wives, but Math arranged for him to marry Podagra, the girl his father wanted him to marry.

That cry, "Lo, Agha, Rhia ith mine," was heard and repeated later all over the estate, the other servants teasing Math with it. He and Rhia had to hear it all day long, repeated with different intonations and even sung to a popular tune. Finally the sentence became contracted to lo-'ga-rhi'th-mi, and so one day, Rhia who was very proud of her husband, suggested that the exponents which he computed be called logarithmi, in honor of that call that brought Math happiness and fame. Later scholars, ignorant of the origin of this name, assumed the final 'i' to be a Latin plural and so it was that for centuries a single exponent became known as the logarithmus. Before presenting his discovery to the Astrologers' Academy, Math wanted a name for the part of the logarithm after the decimal point.

"What was it that you missed the most during those twenty-eight days when you were in the doghouse?" Rhia asked coyly, expecting a sentimental answer.

But Math was truthful, and besides you do not have to be nice to your own wife, so he said, "What I missed the

most was those manti-issas, especially when I could smell them from the kitchen.”

“All right,” said Rhia in an offended tone, “then you are welcome to call the fractional portions of logarithms mantissas, so you will never miss those nasty things again.”

And this is how we still refer to mantissas of logarithms.

Moral of the story: “Don’t pick out a girl for whom you have to do a lot of arithmetic and algebra. There are plenty of others, just as good, for whom you don’t have to lift a finger.”